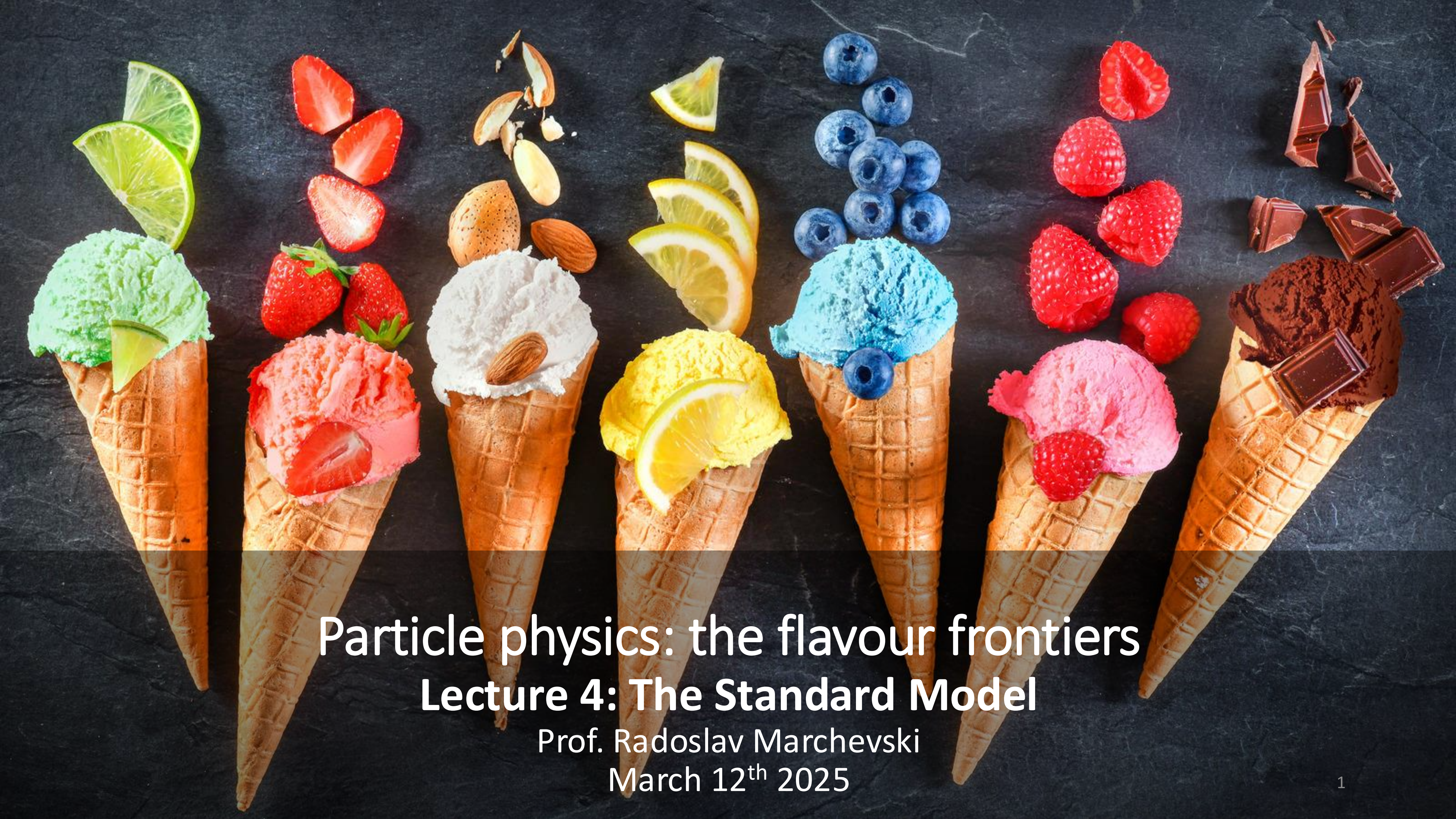

Particle physics: the flavour frontiers

Lecture 4: The Standard Model

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Short recap

Last time we discussed

- Examples and characteristics of non-Abelian symmetries (global and local) and how they give rise to vector boson self-interactions
- Mechanism of spontaneous symmetry breaking, generating masses of scalar, vector, and fermions fields

Today's learning targets

Today you will …

- learn about the Standard Model of particle physics
    - obtain the spectrum of physically observable particles and the possible interactions
    - mass generation mechanism through Spontaneous Symmetry Breaking (SBB)
    - origin of flavour changing interactions in the SM

The Standard Model (SM)

- The Standard Model of particle physics accounts for the strong, weak, and electromagnetic interactions
- We will obtain its spectrum, interactions, accidental symmetries, and parameters

1. The symmetry is local

$$SU(3)_C \times\ SU(2)_L \times\ U(1)_Y$$

2. There are three fermion generations, each consisting of five different representations

$$Q_{Li}(3,2)_{+1/6}, \qquad U_{Ri}(3,1)_{+2/3}, \qquad D_{Ri}(3,1)_{-1/3}, \qquad L_{Li}(1,2)_{-1/2}, \qquad E_{Ri}(1,1)_{-1}, \qquad i = 1,2,3$$

3. There is a single scalar multiplet

$$\phi(1,2)_{+1/2}$$

4. The pattern of spontaneous symmetry breaking (SBB) is

$$SU(3)_C \times\ SU(2)_L \times\ U(1)_Y \rightarrow SU(3)_C \times\ U(1)_{\mathrm{EM}}$$

The Standard Model (SM)

quarks

$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3},$

leptons

$L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1},$

$i = 1,2,3$

Higgs field

$\phi(1,2)_{+1/2}$

- *Quarks:* triplets under $SU(3)_C$
- Leptons: singlets under $SU(3)_C$
- *Higgs field:* singlet under $SU(3)_C$

- The most general renormalizable Lagrangian with scalar and fermion fields is

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\psi} + \mathcal{L}_{\phi} + \mathcal{L}_{\text{Yuk}}$$

- No fermion terms are allowed $\mathcal{L}_{\psi} = 0$. Why?

$\mathcal{L}_{\mathbf{kin}}$

- The SM group has 12 generators
  - 8 L_as that form the $SU(3)$ algebra
  - 3 T_bs that form the $SU(2)$ algebra
  - single Y that form the $U(1)$ algebra

$$[L_a, L_b] = i f_{abc} L_c$$

$$[T_a, T_b] = i\epsilon_{abc} T_c$$

$$[L_a, T_b] = [L_a, Y] = [T_b, Y] = 0$$

$L_a = \frac{\lambda_a}{2}$ (Gell-Mann matrices)

$T_b = \frac{\sigma_b}{2}$ (Pauli matrices)

- The 12 generators correspond to 12 gauge boson degrees of freedom

$$G_a^\mu(8,1)_0, \qquad W_a^{\mu\nu}(1,3)_0, \qquad B^\mu(1,1)_0$$

- With the corresponding field strengths and covariant derivative

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu$$

$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\epsilon_{abc} W_b^\mu W_c^\nu$$

$$B^{\mu\nu} = \partial^\mu B_a^\nu - \partial^\nu B_a^\mu$$

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' Y B^\mu$$

$\mathcal{L}_{\textbf{kin}}$

- Explicitly the covariant derivatives acting of the various scalar and fermion fields are:

$$D^{\mu}\phi = \left(\partial^{\mu} + \frac{i}{2} g W_b^{\mu} \sigma_b + \frac{i}{2} g' B^{\mu}\right)\phi$$

$$D^{\mu}Q_L = \left(\partial^{\mu} + \frac{i}{2} g_s G_a^{\mu} \lambda_a + \frac{i}{2} g W_b^{\mu} \sigma_b + \frac{i}{6} g' B^{\mu}\right) Q_L$$

$$D^{\mu}U_R = \left(\partial^{\mu} + \frac{i}{2} g_s G_a^{\mu} \lambda_a + \frac{2i}{3} g' B^{\mu}\right) U_R$$

$$D^{\mu}D_R = \left(\partial^{\mu} + \frac{i}{2} g_s G_a^{\mu} \lambda_a - \frac{i}{3} g' B^{\mu}\right) D_R$$

$$D^{\mu}L_L = \left(\partial^{\mu} + \frac{i}{2} g W_b^{\mu} \sigma_b - \frac{i}{2} g' B^{\mu}\right) L_L$$

$$D^{\mu}E_R = (\partial^{\mu} - i g' B^{\mu}) E_R$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} G_a^{\mu\nu} G_{a\,\mu\nu} - \frac{1}{4}\, W_b^{\mu\nu} W_{b\,\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + i\overline{Q_{Li}}\gamma_{\mu} D^{\mu} Q_{Li} + i\overline{U_{Ri}}\gamma_{\mu} D^{\mu} U_{Ri} + \overline{D_{Ri}}\gamma_{\mu} D^{\mu} D_{Ri} +$$

$$+\, i\overline{L_{Li}}\gamma_{\mu} D^{\mu} L_{Li} + i\overline{E_{Ri}}\gamma_{\mu} D^{\mu} E_{Ri} + (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi)$$

$\mathcal{L}_{\boldsymbol{\phi}}$ and SSB

$$-\mathcal{L}_\phi = \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2, \qquad v^2 = -\frac{\mu^2}{\lambda}, \qquad \lambda > 0, \mu^2 < 0$$

- *SSB pattern:* $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{EM}$
- ϕ is a singlet under $SU(3)_C$ subgroup, which remains unbroken
- The unbroken subgroup $U(1)_{EM}$ has a generator Q which is identified as $Q = Y + T_3$
- Useful to define $\tilde{\phi}_a = \epsilon_{ab}\phi_b^*$, which transforms as $(1,2)_{-1/2}$

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad \langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \qquad \langle\tilde{\phi}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}$$

$\mathcal{L}_{\mathbf{Yuk}}$

- The Yukawa part of the Lagrangian is given by

$$-\mathcal{L}_{\text{Yuk}} = Y_{ij}^{u}\overline{Q_{Li}}U_{Rj}\tilde{\phi} + Y_{ij}^{d}\overline{Q_{Li}}D_{Rj}\phi + Y_{ij}^{e}\overline{L_{Li}}E_{Rj}\phi + \text{h. c}$$

- $i, j = 1, 2, 3$ represent flavour indexes
- Y_{ij}^{u}, Y_{ij}^{d}, and Y_{ij}^{e} are general complex 3×3 matrices of dimensionless couplings
- Note that the coupling of U_R involve $\tilde{\phi}$
- Fermions in the SM are chiral and charged under $SU(2)_L \times U(1)_Y$ and fermion masses can arise from the Yukawa interactions only as a result of SSB

Spectrum

- *Scalars:* one real scalar field, h, which is electromagnetically neutral ($q = 0$)
  - $m_h = \sqrt{2\lambda}v$
  - Experimentally: $m_h = 125.25 \pm 0.17$ GeV
- *Vector bosons:* three out of four generators spontaneously broken → three out of the four bosons acquire masses

$$\mathcal{L}_{M_V} = (D^\mu\langle\phi\rangle)^\dagger\left(D_\mu\langle\phi\rangle\right)$$

$$D^\mu\langle\phi\rangle = \frac{i}{\sqrt{8}}\left(gW_a^\mu\sigma_a + g'B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix} = \frac{iv}{\sqrt{8}}\begin{pmatrix}g\left(W_1^\mu - iW_2^\mu\right)\\ -gW_3^\mu + g'B^\mu\end{pmatrix}$$

$$\mathcal{L}_{M_V} = \frac{v^2}{\sqrt{8}}\left(g(W_1 + iW_2)_\mu \quad , -gW_{3\mu} + g'B_\mu\right)\begin{pmatrix}g\left(W_1^\mu - iW_2^\mu\right)\\ -gW_3^\mu + g'B^\mu\end{pmatrix}$$

Spectrum

- *Vector bosons:* three out of four generators spontaneously broken → three out of the four bosons acquire masses
- We define the angle θ_W via $\tan\theta_W \equiv g'/g$
- Four gauge boson fields

complex field

real fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)_\mu, \qquad Z_\mu^0 = \cos\theta_W\, W_{3\mu} - \sin\theta_W\, B_\mu, \qquad A_\mu^0 = \sin\theta_W\, W_{3\mu} + \cos\theta_W\, B_\mu$$

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2 v^2 W^{+\mu} W_\mu^- + \frac{1}{8}(g^2 + g'^2)v^2 Z^\mu Z_\mu$$

$$m_W^2 = \frac{1}{4}g^2 v^2, \qquad m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2, \qquad m_A^2 = 0$$

- Charges under the unbroken $U(1)_{\text{EM}}$ symmetry

$$q\left(W_\mu^+\right) = +1, \qquad q\left(W_\mu^-\right) = -1, \qquad q\left(Z_\mu^0\right) = q\left(A_\mu^0\right) = 0,$$

Spectrum

- *Points worth emphasizing:*
    - three vector bosons acquire masses
    - $m_A^2 = 0$ provides a consistency check
    - when a symmetry is partially broken, mass eigenstates can be a linear combination of states from different representations as long as they transform in the same way under the unbroken symmetry
    - the angle θ_W represents a rotation angle from the interaction basis to the mass basis
- SSB leads to relations between observables that would have been independent in the absence of symmetry

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2}$$

$$m_W = 80.377 \pm 0.012 \text{ GeV}$$

$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

- This relation is testable by measuring the masses of the bosons and their couplings and can be expressed as

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$$

Spectrum

- *Fermions:* the terms that lead to the quark and lepton masses can arise only from the Yukawa part of $\mathcal{L}$

$$-\mathcal{L}_{\mathrm{M}_f} = Y_{ij}^{u}\overline{Q_{Li}}U_{Rj}\langle\tilde{\phi}\rangle + Y_{ij}^{d}\overline{Q_{Li}}D_{Rj}\langle\phi\rangle + Y_{ij}^{e}\overline{L_{Li}}E_{Rj}\langle\phi\rangle + \mathrm{h.c}$$

$$Q_L^i = \begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix}, \qquad L_L^i = \begin{pmatrix} N_L^i \\ E_L^i \end{pmatrix}, \qquad U_R^i, \qquad D_R^i, \qquad E_R^i$$

- The quark and lepton eigenstates are in a generic basis

- The various states have well-defined charges under the unbroken $U(1)_{\mathrm{EM}}$ symmetry
  - U_L^i have $T_3 = +1/2$ and $Y = +1/6$, while U_R^i have $T_3 = 0$ and $Y = +2/3$ leading to: $q(U_L^i) = q(U_R^i) = +2/3$
  - D_L^i have $T_3 = -1/2$ and $Y = +1/6$, while D_R^i have $T_3 = 0$ and $Y = -1/3$ leading to: $q(U_L^i) = q(U_R^i) = -1/3$
  - E_L^i have $T_3 = -1/2$ and $Y = -1/2$, while U_R^i have $T_3 = 0$ and $Y = -1$ leading to: $q(E_L^i) = q(E_R^i) = -1$
  - N_L^i have $T_3 = +1/2$ and $Y = -1/2$, hence they are neutral under the unbroken symmetry: $q(N_L^i) = 0$

- If $\mathrm{SU}(2) \times U(1)_Y$ was exact: no way of distinguishing particles members of the same multiplet (we can after SSB)

Spectrum

- Let’s focus on the quarks

$$-\mathcal{L}_{\mathrm{M_q}} = \frac{Y^u_{ij} v}{\sqrt{2}} \overline{U_{Li}} U_{Rj} + \frac{Y^d_{ij} v}{\sqrt{2}} \overline{D_{Li}} D_{Rj} + \mathrm{h.c}$$

- Without loss of generality we can use a bi-unitary transformation to diagonalize the Yukawa matrices

$$Y^u \to \hat{Y}^u = V_{uL} Y^u V^\dagger_{uR}$$

- and transform the basis into one where $\hat{Y}^u$ is diagonal and real

$$\hat{Y}^u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

- This is the mass basis for up-type quarks $y_u < y_c < y_t$

$$Q_{Lu} = \begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \qquad Q_{Lc} = \begin{pmatrix} c_L \\ d_{cL} \end{pmatrix}, \qquad Q_{Lt} = \begin{pmatrix} t_L \\ d_{tL} \end{pmatrix}, \qquad u_R, \qquad c_R, \qquad t_R$$

Spectrum

- Let's focus on the quarks

$$-\mathcal{L}_{\mathrm{M_u}} = \frac{y_u v}{\sqrt{2}} \overline{u_L} u_R + \frac{y_c v}{\sqrt{2}} \overline{c_L} c_R + \frac{y_t v}{\sqrt{2}} \overline{t_L} t_R + \mathrm{h.c}$$

- We conclude that the up-type quarks u, c, and t are Dirac fermions with masses

$$m_u = \frac{y_u v}{\sqrt{2}}, \qquad m_c = \frac{y_c v}{\sqrt{2}}, \qquad m_t = \frac{y_t v}{\sqrt{2}}$$

- Let's use another bi-unitary transformation

$$Y^d \to \hat{Y}^d = V_{dL} Y^d V_{dR}^\dagger$$

$$\hat{Y}^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

- This is the mass basis for down-type quarks $y_d < y_s < y_b$

$$Q_{Ld} = \binom{u_{dL}}{d_L}, \qquad Q_{Lc} = \binom{c_{dL}}{s_L}, \qquad Q_{Lt} = \binom{u_{bL}}{b_L}, \qquad d_R, \qquad s_R, \qquad b_R$$

Spectrum

$$-\mathcal{L}_{\mathrm{M_u}} = \frac{y_u v}{\sqrt{2}}\overline{u_L}u_R + \frac{y_c v}{\sqrt{2}}\overline{c_L}c_R + \frac{y_t v}{\sqrt{2}}\overline{t_L}t_R + \mathrm{h.c}$$

$$m_u = \frac{y_u v}{\sqrt{2}}, \qquad m_c = \frac{y_c v}{\sqrt{2}}, \qquad m_t = \frac{y_t v}{\sqrt{2}}$$

$$-\mathcal{L}_{\mathrm{M_d}} = \frac{y_d v}{\sqrt{2}}\overline{d_L}d_R + \frac{y_s v}{\sqrt{2}}\overline{s_L}s_R + \frac{y_b v}{\sqrt{2}}\overline{b_L}b_R + \mathrm{h.c}$$

$$m_d = \frac{y_d v}{\sqrt{2}}, \qquad m_s = \frac{y_s v}{\sqrt{2}}, \qquad m_b = \frac{y_b v}{\sqrt{2}}$$

- All charged fermions acquire Dirac masses as a result of the SSB!
- **Key point:** the charged fermions are in chiral representation of the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group but they are in vector like representation of the broken $SU(3)_C \times U(1)_{\mathrm{EM}}$ group

Spectrum

- **Key point:** the charged fermions are in chiral representation of the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group but they are in vector like representation of the broken $SU(3)_C \times U(1)_{\mathrm{EM}}$ group
  - left-handed and right-handed charged lepton fields e, μ, τ are in the $(1)_{-1}$ representation
  - left-handed and right-handed up-type quark fields u, c, t are in the $(3)_{+2/3}$ representation
  - left-handed and right-handed down-type quark fields d, s, b are in the $(3)_{-1/3}$ representation
  - neutrinos remain massless $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$
- Experimentally the values of the charged fermion masses are

$$m_e = 0.51099895000(15)\ \mathrm{MeV}, \qquad m_\mu = 105.6583755(23)\ \mathrm{MeV}, \qquad m_\tau = 1776.86(12)\ \mathrm{MeV}$$

$$m_u = 2.2^{+0.5}_{-0.3}\ \mathrm{MeV}, \qquad m_c = 1.27 \pm 0.02\ \mathrm{GeV}, \qquad m_t = 172.7 \pm 0.3\ \mathrm{GeV}$$

$$m_d = 4.7^{+0.5}_{-0.2}\ \mathrm{MeV}, \qquad m_s = 93^{+9}_{-3}\ \mathrm{MeV}, \qquad m_b = 4.18^{+0.03}_{-0.02}\ \mathrm{GeV}$$

The issue of how quark masses are defined and extracted is complicated by the fact that QCD is confining

The CKM matrix

- Important difference between the quark and lepton spectrum
  - for the leptons there exist a basis that is simultaneously an interaction and a mass basis for both the charged and neutral leptons
  - for the quarks there is no interaction basis that is also a mass basis for both up- and down-type quarks
- We denote $u^i = (u, c, t)$ and $d^i = (d, s, b)$ and write the relation of the mass eigenstates to the interaction eigenstates

$$u_L^i = (V_{uL})_{ij} U_L^j, \qquad u_R^i = (V_{uR})_{ij} U_R^j, \qquad d_L^i = (V_{dL})_{ij} D_L^j, \qquad d_R^i = (V_{dR})_{ij} D_R^j$$

$V_{uL} \neq V_{dL}$

$$-\mathcal{L}_{\mathrm{M_u}} = \frac{y_u v}{\sqrt{2}} \overline{u_L} u_R + \frac{y_c v}{\sqrt{2}} \bar{c}_L c_R + \frac{y_t v}{\sqrt{2}} \bar{t}_L t_R + \mathrm{h.c} \qquad Y^u = \hat{Y}^u, \qquad Y^d = V \hat{Y}^d$$

different bases

$$-\mathcal{L}_{\mathrm{M_d}} = \frac{y_d v}{\sqrt{2}} \overline{d_L} d_R + \frac{y_s v}{\sqrt{2}} \bar{s}_L s_R + \frac{y_b v}{\sqrt{2}} \overline{b_L} b_R + \mathrm{h.c} \qquad Y^d = \hat{Y}^d, \qquad Y^u = V^\dagger \hat{Y}^u$$

The CKM matrix

- The four matrices V_{uL}, V_{uR}, V_{dL}, and V_{dR} depend on the basis from which we start the diagonalization
- The combination $\mathrm{V} = V_{uL}V_{dL}^{\dagger}$ does not $\Longrightarrow$ indication that V is physical!
- The V matrix is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$u_L^i = (V_{uL})_{ij}U_L^j, \qquad u_R^i = (V_{uR})_{ij}U_R^j, \qquad d_L^i = (V_{dL})_{ij}D_L^j, \qquad d_R^i = (V_{dR})_{ij}D_R^j$$

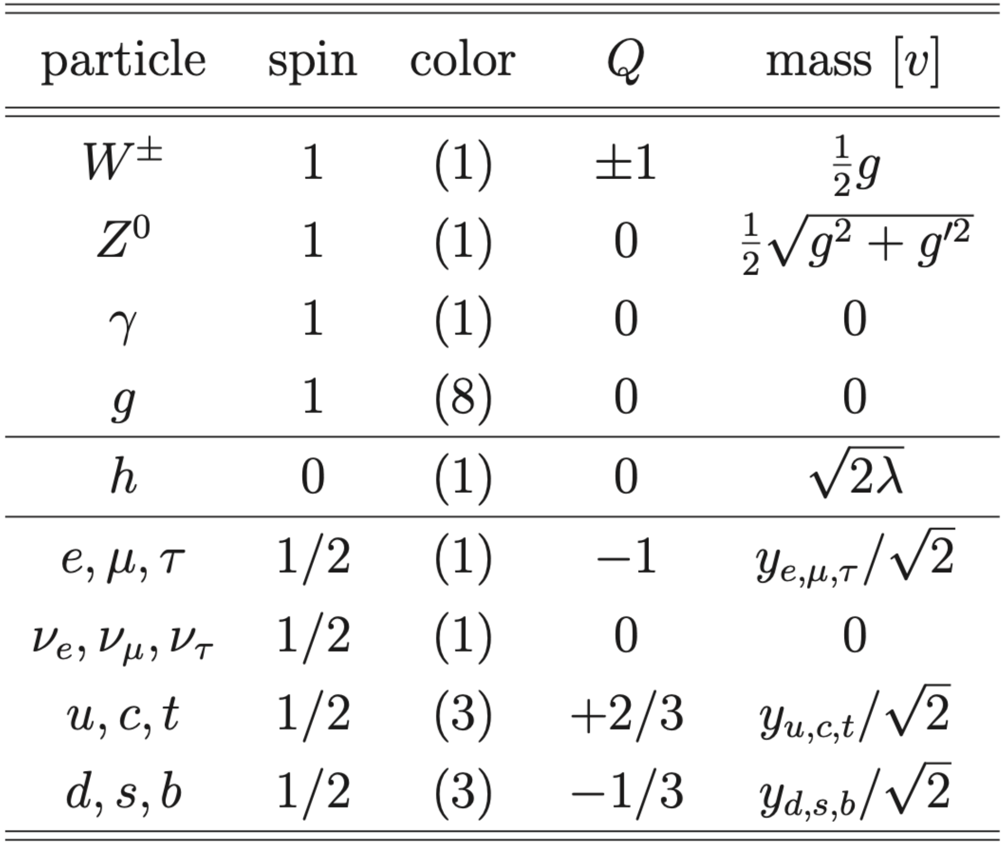
$$V_{uL} \neq V_{dL}$$

different bases

$$-\mathcal{L}_{\mathrm{M_u}} = \frac{y_u v}{\sqrt{2}}\overline{u_L}u_R + \frac{y_c v}{\sqrt{2}}\overline{c_L}c_R + \frac{y_t v}{\sqrt{2}}\overline{t_L}t_R + \mathrm{h.c} \qquad Y^u = \hat{Y}^u, \qquad Y^d = V\hat{Y}^d$$

$$-\mathcal{L}_{\mathrm{M_d}} = \frac{y_d v}{\sqrt{2}}\overline{d_L}d_R + \frac{y_s v}{\sqrt{2}}\overline{s_L}s_R + \frac{y_b v}{\sqrt{2}}\overline{b_L}b_R + \mathrm{h.c} \qquad Y^d = \hat{Y}^d, \qquad Y^u = V^{\dagger}\hat{Y}^u$$

Particles (mass eigenstates)



particle	spin	color	Q	mass $[v]$
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2 + g'^2}$
γ	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e, μ, τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e, ν_μ, ν_τ	1/2	(1)	0	0
u, c, t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d, s, b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

- Mass eigenstates of the Standard Model, their $\mathrm{SU}(3) \times U(1)_{\mathrm{EM}}$ quantum numbers, and their masses in units of the VEV v
- All masses are proportional to the VEV of the scalar field, v → result of SSB

Electromagnetic (QED) and Strong (QCD) interactions

- Photon-mediated electromagnetic interaction are described by Quantum Electro-Dynamics (QED), part of the SM
- We get the following Lagrangian terms for the interaction of the photon field with the SM Dirac fermions

$$\mathcal{L}_{Af\bar{f}} = e\overline{e}_i\gamma^{\mu}A_{\mu}e_i - \left(\frac{2}{3}\right)e\overline{u}_i\gamma^{\mu}A_{\mu}u_i + \left(\frac{1}{3}\right)e\overline{d}_i\gamma^{\mu}A_{\mu}d_i$$

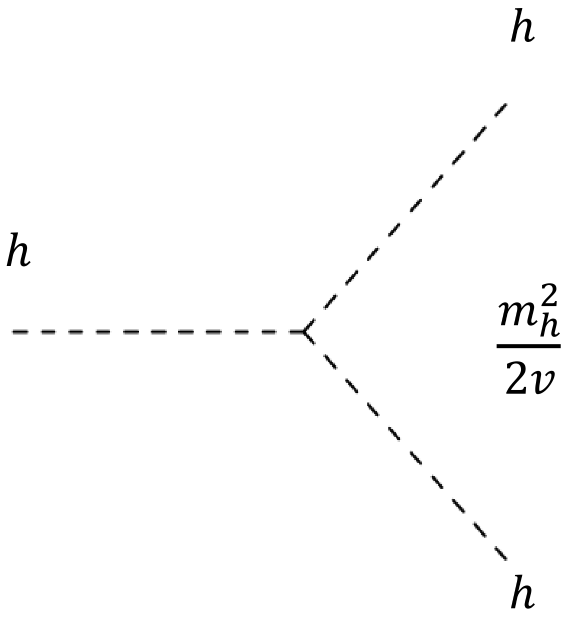
- The gluon-mediated strong interactions are described by QCD, which is part of the SM
- We get the following Lagrangian terms for the interaction of the gluon field with the SM quarks

$$\mathcal{L}_{Gq\bar{q}} = -g_s\overline{q}_i\gamma^{\mu}G_{a\,\mu}\left(\frac{\lambda_a}{2}\right)q_i$$

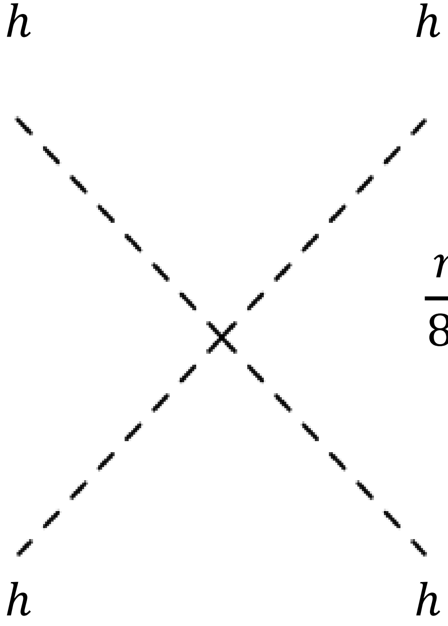
The Higgs Boson interactions

$$\mathcal{L}_{\text{int}}^{h} = \mathcal{L}_{h}^{h} + \mathcal{L}_{V}^{h} + \mathcal{L}_{f}^{h}$$

$$-\mathcal{L}_{h}^{h} = \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4$$



$$\frac{m_h^2}{2v} = \lambda v$$

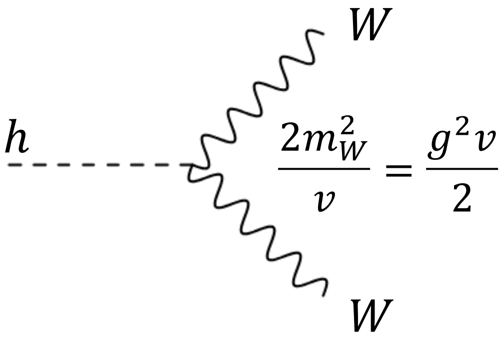


$$\frac{m_h^2}{8v^2} = \frac{\lambda}{4}$$

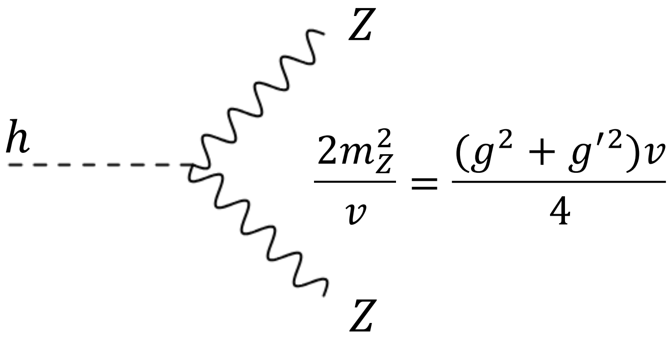
The Higgs Boson interactions

$$\mathcal{L}_{\text{int}}^{h} = \mathcal{L}_{h}^{h} + \mathcal{L}_{V}^{h} + \mathcal{L}_{f}^{h}$$

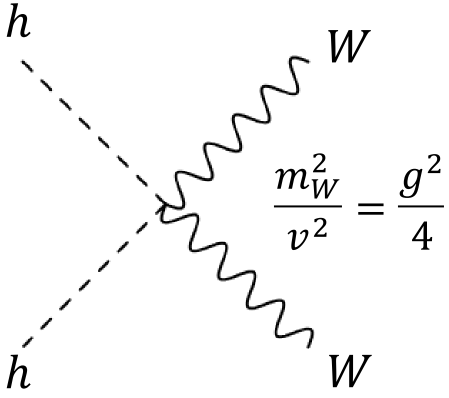
$$-\mathcal{L}_{V}^{h} = \left(\frac{2h}{v} + \frac{h^2}{v^2}\right)\left(m_W^2 W_\mu^- W^{\mu+} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$



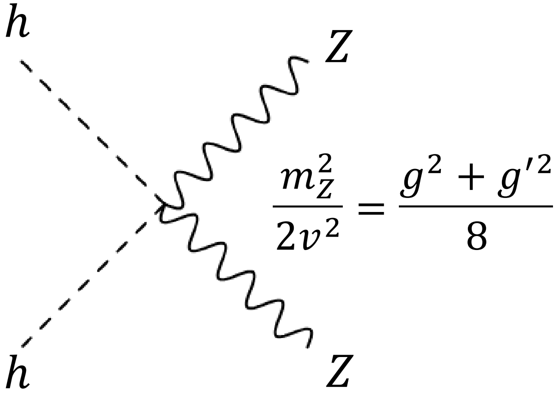
$$\frac{2m_W^2}{v} = \frac{g^2 v}{2}$$



$$\frac{2m_Z^2}{v} = \frac{(g^2 + g'^2)v}{4}$$



$$\frac{m_W^2}{v^2} = \frac{g^2}{4}$$

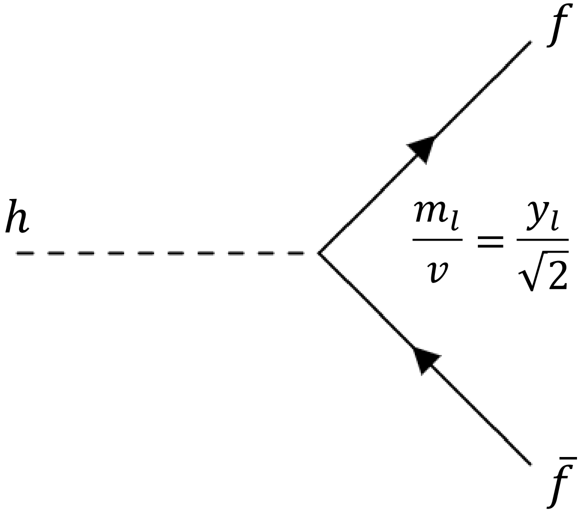


$$\frac{m_Z^2}{2v^2} = \frac{g^2 + g'^2}{8}$$

The Higgs Boson interactions

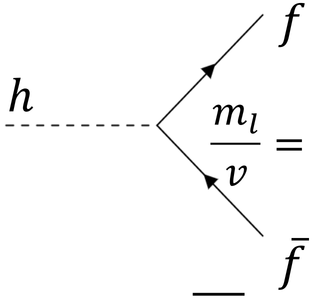
$$\mathcal{L}_{\text{int}}^{h} = \mathcal{L}_{h}^{h} + \mathcal{L}_{V}^{h} + \mathcal{L}_{f}^{h}$$

$$-\mathcal{L}_{f}^{h} = \frac{h}{v}\left(m_e \overline{e_L} e_R + m_\mu \overline{\mu_L} \mu_R + m_\tau \overline{\tau_L} \tau_R + m_u \overline{u_L} u_R + m_c \overline{c_L} c_R + m_t \overline{t_L} t_R + m_d \overline{d_L} d_R + m_s \overline{s_L} s_R + m_b \overline{b_L} b_R + \text{h.c.}\right)$$



The Higgs Boson interactions

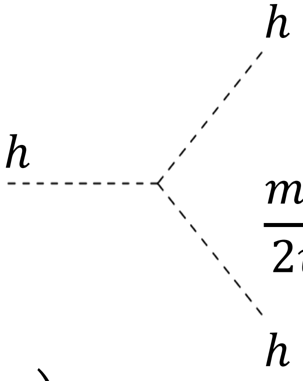
$$\mathcal{L}_{\text{int}}^{h} = \mathcal{L}_{h}^{h} + \mathcal{L}_{V}^{h} + \mathcal{L}_{f}^{h}$$



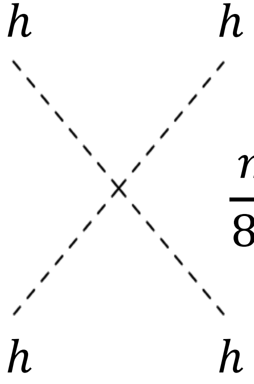
$$\frac{m_l}{v} = \frac{y_l}{\sqrt{2}}$$

$$-\mathcal{L}_{f}^{h} = \frac{h}{v}\left(m_e\overline{e_L}e_R + m_\mu\overline{\mu_L}\mu_R + m_\tau\overline{\tau_L}\tau_R + m_u\overline{u_L}u_R + m_c\overline{c_L}c_R + m_t\overline{t_L}t_R + m_d\overline{d_L}d_R + m_s\overline{s_L}s_R + m_b\overline{b_L}b_R + \text{h.c.}\right)$$

$$-\mathcal{L}_{h}^{h} = \frac{m_h^2}{2v}h^3 + \frac{m_h^2}{8v^2}h^4$$

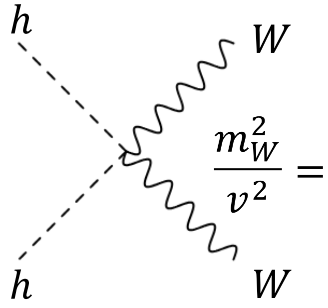


$$\frac{m_h^2}{2v} = \lambda v$$

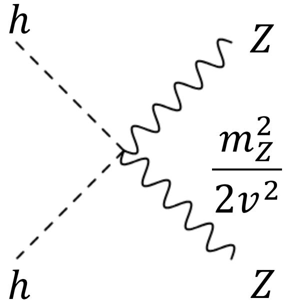


$$\frac{m_h^2}{8v^2} = \frac{\lambda}{4}$$

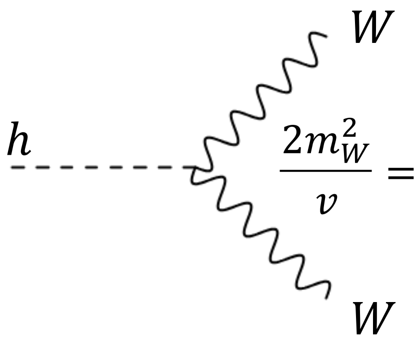
$$-\mathcal{L}_{V}^{h} = \left(\frac{2h}{v} + \frac{h^2}{v^2}\right)\left(m_W^2 W_\mu^- W^{\mu+} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right)$$



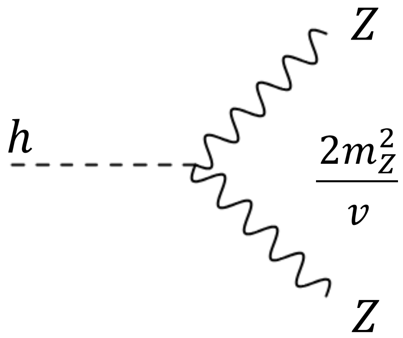
$$\frac{m_W^2}{v^2} = \frac{g^2}{4}$$



$$\frac{m_Z^2}{2v^2} = \frac{g^2 + g'^2}{8}$$



$$\frac{2m_W^2}{v} = \frac{g^2 v}{2}$$

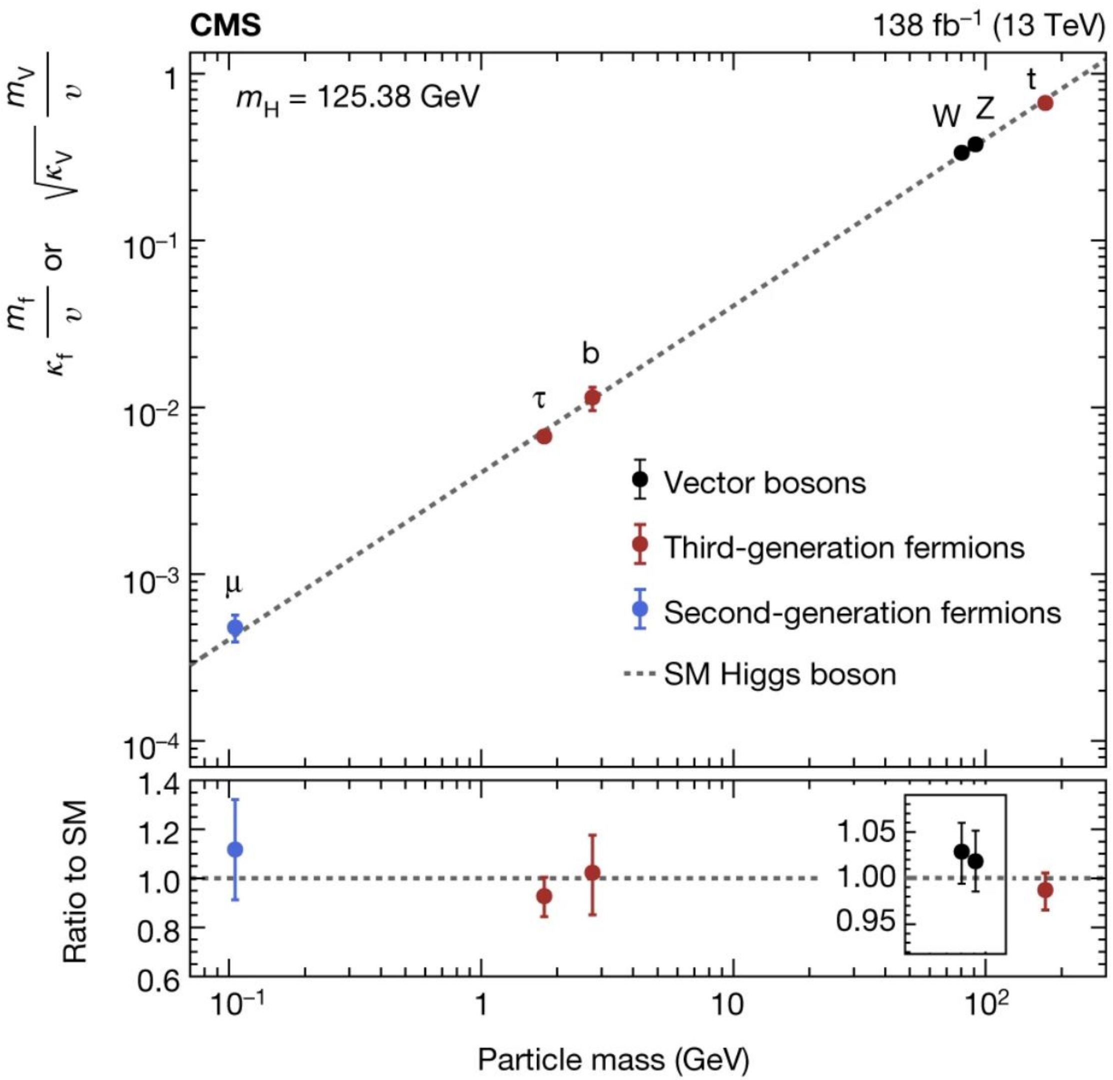


$$\frac{2m_Z^2}{v} = \frac{(g^2 + g'^2)v}{4}$$

All Higgs couplings are proportional to the mass of the particle to which it couples

The Higgs Boson interactions

- Higgs decays are proportional to the mass of the particle to which it couples
- The Higgs decay is dominated by the heaviest particle that can be pair-produced in the decay
- Decays to $t\bar{t}, \gamma\gamma, gg$ are not possible at tree level and can only happen via loops
- Not all Higgs decays are experimentally established
  - at present only ($\mu^+\mu^-$, 3σ so far), $\tau^+\tau^-, b\bar{b}, ZZ^\star, WW^*, \gamma\gamma$ are established with rates consistent with the SM predictions



$$BR_{b\bar{b}}: BR_{WW^*}: BR_{gg}: BR_{\tau^+\tau^-}: BR_{ZZ^*}: BR_{c\bar{c}} = 0.58: 0.21: 0.09: 0.06: 0.03: 0.03$$

WW^* and $ZZ^*$are decays with one boson on-shell and the other off-shell

Neutral current weak interactions

- Z boson couplings to fermions is proportional to

$$g c_W T_3 - g' s_W Y = \frac{\mathrm{g}}{c_W}(T_3 - s_W^2 Q), \qquad c_W = \cos\theta_W\,, \qquad s_W = \sin\theta_W$$

- Using the T_3 and Y assignments of the various fermion fields, we find the following types of Z couplings

$$\mathcal{L}_{Zf\bar{f}} = \frac{g}{c_W}\Big(-\left(\frac{1}{2} - s_W^2\right)\overline{e_L^i}\gamma^\mu Z_\mu e_L^i + s_W^2\overline{e_R^i}\gamma^\mu Z_\mu e_R^i + \frac{1}{2}\overline{\nu_L^i}\gamma^\mu Z_\mu \nu_L^i + \left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\overline{u_L^i}\gamma^\mu Z_\mu u_L^i -$$

$$\frac{2}{3}s_W^2\overline{u_R^i}\gamma^\mu Z_\mu u_R^i - \left(\frac{1}{2} - \frac{1}{3}s_W^2\right)\overline{d_L^i}\gamma^\mu Z_\mu d_L^i + \frac{1}{3}s_W^2\overline{d_R^i}\gamma^\mu Z_\mu d_R^i\Big)$$

Important features of neutral-current weak interatcions (NCWI):

? ? ?

Neutral current weak interactions

- Z boson couplings to fermions is proportional to

$$g c_W T_3 - g' s_W Y = \frac{\mathrm{g}}{c_W}(T_3 - s_W^2 Q), \qquad c_W = \cos\theta_W\,, \qquad s_W = \sin\theta_W$$

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$$\mathcal{L}_{Zf\bar{f}} = \frac{g}{c_W}\Big(-\left(\tfrac{1}{2} - s_W^2\right)\overline{e_L^i}\gamma^\mu Z_\mu e_L^i + s_W^2\overline{e_R^i}\gamma^\mu Z_\mu e_R^i + \tfrac{1}{2}\overline{\nu_L^i}\gamma^\mu Z_\mu \nu_L^i + \left(\tfrac{1}{2} - \tfrac{2}{3}s_W^2\right)\overline{u_L^i}\gamma^\mu Z_\mu u_L^i -$$

$$\tfrac{2}{3}s_W^2\overline{u_R^i}\gamma^\mu Z_\mu u_R^i - \left(\tfrac{1}{2} - \tfrac{1}{3}s_W^2\right)\overline{d_L^i}\gamma^\mu Z_\mu d_L^i + \tfrac{1}{3}s_W^2\overline{d_R^i}\gamma^\mu Z_\mu d_R^i\Big)$$

Important features of neutral-current weak interatcions (NCWI):

- Z boson couples to neutrinos
- *Parity violation:* Z boson couplings are chiral (LH and RH fields carry different T_3, Z couples to them differently)
- *Diagonality:* the Z boson couple for example to e^+e^-, $\mu^+\mu^-$, $\overline{\nu_{\tau L}}\nu_{\tau L}$, $\overline{\nu_{\mu L}}\nu_{\mu L}$ but not to $e^\pm\mu^\mp$, $\overline{\nu_{\mu L}}\nu_{\tau L}$ pairs
- *Universality:* the couplings of the Z boson to the different generations are universal

Neutral current weak interactions

Predictions (omitting common
and phase space factors)

$$\Gamma(Z \to \nu\bar{\nu}) \propto 1$$

$$\Gamma\left(Z \to l\bar{l}\right) \propto 1 - 4s_W^2 - 8s_W^4$$

$$\Gamma(Z \to u\bar{u}) \propto 3\left(1 - \frac{8}{3}s_W^2 - \frac{32}{9}s_W^4\right)$$

$$\Gamma\left(Z \to d\bar{d}\right) \propto 3\left(1 - \frac{4}{3}s_W^2 - \frac{8}{9}s_W^4\right)$$

$s_W^2 = 0.225$

$$\Gamma_\nu : \Gamma_l : \Gamma_u : \Gamma_d = 1 : 0.51 : 1.74 : 2.24$$

Experimental values

$$BR(Z \to \nu\bar{\nu}) = (6.67 \pm 0.02)\%$$

$$BR\left(Z \to l\bar{l}\right) = (3.366 \pm 0.002)\%$$

$$BR\ (Z \to u\bar{u}) = (11.6 \pm 0.6)\%$$

$$BR\ \left(Z \to d\bar{d}\right) = (15.6 \pm 0.4)\%$$

$$\Gamma_\nu : \Gamma_l : \Gamma_u : \Gamma_d = 1 : 0.505 : 1.74 : 2.34$$

Charged current weak interactions

- Charged $W^{\pm}$ boson couplings to fermions are simple
  - the interaction basis is also mass basis → the W interactions are universal

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^{\mu}W_{\mu}^{+}e_{L}^{-} + \overline{\nu_{\mu L}}\gamma^{\mu}W_{\mu}^{+}\mu_{L}^{-} + \overline{\nu_{\tau L}}\gamma^{\mu}W_{\mu}^{+}\tau_{L}^{-} + \mathrm{h.c.}\right)$$

- More complicated in the quark sector
  - no interaction basis that is also a mass basis

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{U_L^i}\gamma^{\mu}W_{\mu}^{+}D_L^i + \mathrm{h.c.}$$

$$U_L^i = \left(V_{uL}^{\dagger}\right)_{ij}u_L^j, \qquad D_L^i = \left(V_{dL}^{\dagger}\right)_{ij}d_L^j$$

$$\Downarrow$$

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{u_L^k}\left(V_{uL}\right)_{ki}\gamma^{\mu}W_{\mu}^{+}\left(V_{dL}^{\dagger}\right)_{il}d_L^l + \mathrm{h.c.} = -\frac{g}{\sqrt{2}}\overline{u_L^k}\underset{\substack{\uparrow \\ \text{CKM matrix}}}{V_{kl}}\gamma^{\mu}W_{\mu}^{+}d_L^l + \mathrm{h.c.}$$

Charged current weak interactions

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^{\mu}W_{\mu}^{+}e_{L}^{-} + \overline{\nu_{\mu L}}\gamma^{\mu}W_{\mu}^{+}\mu_{L}^{-} + \overline{\nu_{\tau L}}\gamma^{\mu}W_{\mu}^{+}\tau_{L}^{-} + \mathrm{h.c.}\right)$$

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}\left(V_{uL}\right)_{ki}\gamma^{\mu}W_{\mu}^{+}\left(V_{dL}^{\dagger}\right)_{il}d_{L}^{l} + \mathrm{h.c.} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}V_{kl}\gamma^{\mu}W_{\mu}^{+}d_{L}^{l} + \mathrm{h.c.}$$

Important features of charged-current weak interactions (CCWI):

???

Charged current weak interactions

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^{\mu}W_{\mu}^{+}e_{L}^{-} + \overline{\nu_{\mu L}}\gamma^{\mu}W_{\mu}^{+}\mu_{L}^{-} + \overline{\nu_{\tau L}}\gamma^{\mu}W_{\mu}^{+}\tau_{L}^{-} + \text{h. c.}\right)$$

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}\left(V_{uL}\right)_{ki}\gamma^{\mu}W_{\mu}^{+}\left(V_{dL}^{\dagger}\right)_{il}d_{L}^{l} + \text{h. c.} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}V_{kl}\gamma^{\mu}W_{\mu}^{+}d_{L}^{l} + \text{h. c.}$$

Important features of charged-current weak interactions (CCWI):

- *Parity violation:* only left-handed particles participate in CCWI
- The W couplings to quark mass eigenstates are **NOT universal** the universality of gauge interactions is hidden in the unitarity of the CKM matrix, V
- The W couplings to the quark eigenstates are **NOT diagonal**. This is a consequence of the fact that no pair of up and down type mass eigenstates fits into an $SU(2)$ doublet.

Summary of Lecture 4

Main learning outcomes

- The spectrum and interactions described by the Standard Model of particle physics
- Mass generation mechanism through Spontaneous Symmtery Breaking
- How flavour arises in the Standard Model